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Speculation in Financial Markets: A Survey.

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Abstract

This survey covers the microeconomic theory of speculation in financial markets, since the development of the economics of uncertainty. It starts with a description of Walrasian exchange economies, both in general equilibrium —the Arrow-Debreu model and its extensions— and in partial equilibrium. Speculation, it is explained, is an incomplete-market phenomenon. It proceeds by analyzing more general voluntary trade environments, with a focus on whether or not differences in information are a valid source for belief heterogeneity. The role of common priors in the no-trade theorem is discussed. Finally, heterogeneous priors models are considered.

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1 Introduction

To speculate in asset markets is to trade motivated exclusively by the possibility of capital gains. Speculation, then, involves a belief, possibly divergent from the rest of market participants, that leads the speculator to bet against them. To explain the phenomenon of speculation is to explain under what conditions we are likely to observe belief-based trading.

We start off by analyzing the conditions to observe speculation. These will be conditions on the market structure, and on the sources of belief heterogeneity. Section 2 reviews Walrasian exchange economies, the standard model for competitive environments, where it is established that a pre-condition for the existence of speculation is that the market structure be incomplete. Section 3 discusses the conceptual problems to explain belief heterogeneity on the basis of disparate information, and extends the analysis to more general voluntary trade arrangements, stressing the betting component of speculation. Section 4 is devoted to the developments in the heterogeneous prior paradigm. Section 5 concludes.

The primary focus of this survey, it must be emphasized, is on the microeconomic theory of speculation. This means the discussion of the period preceding the development of the economics of information, with macroeconomic orientation —where the accent is placed on the issue of economic stability/instability— is left aside, as well as the empirical work. We will only cite those results that are relevant to the more recent and formal discussion.

2 Walrasian exchange economies with heterogeneous beliefs

Walrasian theory of markets emphasizes price taking and market taking: all market participants (consumers and producers) regard prices and open markets as given, and behave rationally with respect to those parameters (Makowski and Ostroy, 1995). It is in this context that the present section addresses the problem of behavior under heterogeneous beliefs. In order to focus on informational issues, only an exchange economy is considered.

Debreu (1959) first noticed that extending Walras' model of competitive markets to an intertemporal and uncertain environment could be done simply by refining the definition of commodity. Indeed, by treating a liter of milk today as a different object than a liter of milk tomorrow, and the liter of milk tomorrow if it happens to rain as differently as if it does not —that is, by enlarging the commodity space— one is able to analyze dynamic economies operating under uncertainty. There, he makes the fiction of contingent claims being available as well as open markets for all commodities just as in the certainty case. Thus, there is trading in $L \times T \times \Theta$ markets: L goods, T dates and Θ states. In this view, the demand for assets originates on the desire to move consumption across dates and states, that is, (financial) assets are useful because they allow consumption smoothing to risk-averse individuals.

2.1 The basic framework: the two-period complete-market model

In this section we briefly review the standard theory. Let $i \in I$ denote individuals, $\ell \in L$ (perishable) consumption goods, $t \in \{0,1\}$ time periods, $\theta \in \Theta$ states of nature or complete descriptions of the world in $t = 1$, $\Theta_0 \equiv \Theta \cup \{0\}$, $\boldsymbol{\pi}^i$ prior beliefs over Θ, $w_t^i \in R^{L(\Theta+1)}$ endowments, $u_i(c_\theta^i) \in C^2$ utility functions over consequences, $k \in K$ assets, and q_k the price of an asset k.

Debreu (1959) considers the case where there is complete agreement with regard to the possible events (but not necessarily with respect to their likelihood) and where all information is public. Hence, individuals choose the consumption vector $\mathbf{x}^i$ of L commodities indexed by time t and states θ as to

$$\max_{\{\mathbf{x}^i\}} U^i(\mathbf{x}^i) \text{ subject to } \widehat{\mathbf{p}}_0'(\mathbf{x}_0^i - \boldsymbol{\omega}_0^i) + \sum_{\theta\in\Theta} \widehat{\mathbf{p}}_\theta' \mathbf{d}_\theta^i \leq 0 \tag{1}$$

where $d_{\theta\ell}^i$ is the net purchase of claims to good ℓ in state θ, and thus $0 \leq x_{\theta\ell}^i \leq \omega_{\theta\ell}^i + d_{\theta\ell}^i$ $\forall \theta \in \Theta$, $\ell \in L$. $U^i(\mathbf{x}^i)$ is meant to be the von Neumann-Morgenstern utility function $U^i(\mathbf{x}^i) = \sum_{\theta\in\Theta} \pi_\theta^i u^i(x_{\theta 1}^i, ..., x_{\theta L}^i)$. Thus, the budget constraint can be rewritten as $\widehat{\mathbf{p}}'(\mathbf{x}^i - \mathbf{w}^i) \leq 0$, and the problem becomes

$$\max_{\{\mathbf{x}^i\}} U^i(\mathbf{x}^i) \text{ subject to } \widehat{\mathbf{p}}'(\mathbf{x}^i - \mathbf{w}^i) \leq 0 \tag{2}$$

A striking fact is that this problem does not formally differ at all from the standard problem in consumer theory under certainty. This formulation stresses the fact that the consumer is choosing different bundles of consumption goods by buying a special kind of financial asset called "contingent claim".

The equilibrium is characterized by:

Definition 1 *A **Walrasian equilibrium** for the exchange economy* $\mathcal{E} = \{(\mathbf{w}^i, U^i)_{i \in I}\}$, *is a vector* $(\widehat{\mathbf{p}}, \mathbf{x})$ *such that:*

1. $\mathbf{x}^i \in \{\arg\max U^i(\mathbf{x}^i)$ *subject to* $\widehat{\mathbf{p}}'\mathbf{x}^i \leq \widehat{\mathbf{p}}'\mathbf{w}^i\} \; \forall i \in I$
2. $\sum_{i=1}^{I} x^i_{\theta\ell} = \sum_{i=1}^{I} w^i_{\theta\ell} \; \forall \theta \in \Theta_0, \ell \in L$

Theorem 1 *In this economy, the resulting allocation is Pareto-optimal ex-ante and ex-post irrespective of beliefs.*

This proposition, the first theorem of welfare economics, establishes that *all gains from trade are exploited in one round of trade.* Moreover, if markets were to reopen after the true state is known, no trade would occur since the allocation would still be Pareto-optimal (that is, also Pareto-optimal in an ex-post sense). To see this, one only needs to verify that the gradient vector of every individual's utility function is proportional to each other, both ex-ante and ex-post. Ex-ante, we have that $\nabla U^i(\mathbf{x}^i) \propto \nabla U^j(\mathbf{x}^j)$ because there is a market open for each argument of the utility function. Starting from the first order conditions of (2), we have

$$\left\{\nabla U^i(\mathbf{x}^i) = \lambda^i \widehat{\mathbf{p}} \text{ and } \nabla U^j(\mathbf{x}^j) = \lambda^j \widehat{\mathbf{p}}\right\} \Rightarrow \left\{\frac{\nabla U^i(\mathbf{x}^i)}{\lambda^i} = \widehat{\mathbf{p}} = \frac{\nabla U^j(\mathbf{x}^j)}{\lambda^j}\right\} \tag{3}$$

Ex-post, when uncertainty is resolved, the gradient vectors are going to be modified by a proportional factor $\begin{cases} \frac{1}{\pi_\theta} & \text{if } \theta \text{ occurred} \\ 0 & \text{otherwise} \end{cases}$ by Bayes' rule. If we only consider the part of the vector $\mathbf{x}^i$ that contains consumption of goods in the state that actually materialized, $\mathbf{x}^i_\theta$, we have

$$\left\{\frac{1}{\pi^i_\theta}\nabla U^i(\mathbf{x}^i_\theta) = \frac{1}{\pi^i_\theta}\lambda^i \widehat{\mathbf{p}} \text{ and } \frac{1}{\pi^j_\theta}\nabla U^j(\mathbf{x}^j_\theta) = \frac{1}{\pi^j_\theta}\lambda^j \widehat{\mathbf{p}}\right\} \tag{4}$$

$$\Rightarrow \left\{\frac{\frac{1}{\pi^i_\theta}\nabla U^i(\mathbf{x}^i_\theta)}{\frac{\lambda^i}{\pi^i_\theta}} = \widehat{\mathbf{p}} = \frac{\frac{1}{\pi^j_\theta}\nabla U^j(\mathbf{x}^j_\theta)}{\frac{\lambda^j}{\pi^j_\theta}}\right\}$$

Therefore, the opportunity of trading after uncertainty is resolved cannot improve welfare, and the re-opening of the market will not induce consumers to trade.

What this implies, though, is that —in this economy— forecasting future asset prices is not an issue as there will not be open markets in the future, and —subsequently— there can be no speculation.

In effect, any differences in belief with respect to the likelihood of any particular state happening creates at time 0 a trade of contingent claims directly, that will pay consumption goods if the state materializes, so that reversing the position will not be necessary.

Arrow (1963) considers an otherwise similar economy, except that the securities that can be traded are not contingent claims but "pure securities" (also known as

"Arrow securities"), that is, claims to 1 unit of account in state θ. There is trading in Θ state-claims before the resolution of uncertainty, and in L goods after it. This structure requires consumers to forecast consumption good prices $\mathbf{p}_\theta$ for each state. However, since the only uncertainty refers to the state that will materialize (endowments), it seems natural to assume that everyone agrees as to what prices will prevail *if* state θ occurs[1]. Yet, beliefs differ as to how likely it is that state θ occurs. Thus, each consumer solves

$$\max_{\{\mathbf{x}^i,\widehat{\mathbf{a}}^i\}} U^i(\mathbf{x}^i) \text{ subject to } \mathbf{p}_0'\mathbf{x}_0^i + \widehat{\mathbf{q}}'\widehat{\mathbf{a}}^i \leq \mathbf{p}_0'\boldsymbol{\omega}_0^i \text{ and } \left\{\mathbf{p}_\theta'\mathbf{x}_\theta^i \leq \mathbf{p}_\theta'\mathbf{w}_\theta^i + \widehat{a}_\theta^i\right\}_{\theta\in\Theta} \tag{5}$$

where $\widehat{q}_\theta$ is the price today of a pure asset that pays off in state θ and $\widehat{a}_\theta^i$ is the net purchase by person i of an Arrow security that pays off in state θ. However, it is clear that $\mathbf{p}_\theta'\mathbf{x}_\theta^i = \mathbf{p}_\theta'\mathbf{w}_\theta + \widehat{a}_\theta$ if there is local non-satiation. Then, we have that (5) can be written as

$$\max_{\{\mathbf{x}^i\}} U^i(\mathbf{x}^i) \text{ subject to } \mathbf{p}_0'\mathbf{x}_0^i + \sum_{\theta\in\Theta}\widehat{q}_\theta(\mathbf{p}_\theta'\mathbf{x}_\theta^i - \mathbf{p}_\theta'\mathbf{w}_\theta^i) \leq \mathbf{p}_0'\mathbf{w}_0^i \tag{6}$$

which is equivalent to the contingent claim case when $\widehat{p}_{s\ell} = \widehat{q}_s p_{s\ell}$. This means that the two economies are equivalent, in the sense that the consumption sets that these markets give rise to are the same[2] . It follows that there is no special role for

[1] In Radner's (1972) terminology, expectations are "common". We come back to this issue in section 2.2 below.

[2] Provided that we compare the same equilibrium, that is, we consider prices wich satisfy the above equality.

speculation either. Differences in beliefs as to the likelihood of a particular state happening explain trade, but there is no opportunity of capital gains because there are no price changes.

Finally, if instead of pure securities there were markets for ordinary securities, that is, promises of payment of variable numbers of units of account contingent on the occurrence of particular states, or put another way, bundles of pure securities, matters would not be different as long as we still have complete markets. In effect, if R is the $\Theta \times K$ matrix that contains as columns the state-contingent payoffs of the K assets in the Θ states, and if R is of full rank (the complete-markets condition) then R^{-1} is the matrix specifying the portfolios of ordinary assets required to replicate a pure security for each state. Therefore, the consumer has the same options as before[3].

One should emphasize, however, that although there can be no speculation in this setting, asset markets do offer the opportunity of gambling, in the sense that people bet on the occurrence of particular states every time they choose to "put" a larger consumption bundle or larger units of account on them. Differences of opinion $\boldsymbol{\pi}^i$ give rise to trade, even if there were no other motives. Likewise, asset prices are affected by beliefs and their dispersion.

[3] This implicitly assumes the possibility of unlimited short sales, for no restrictions are put on the sign of the entries in R^{-1}.

2.2 More time periods

The extension of the previous framework to more periods requires a reinterpretation of the concept of state and the modeling of uncertainty. One approach is to continue thinking of a state as a complete history of exogenous events, that is, a list of all exogenous events that have taken place on each period. Events, then, become the partial development of history through time. Many relevant aspects of reality are excluded from the description of a state, though, and have to be included otherwise. For instance, one such important feature of the world which is excluded from the description of uncertainty is the future level of asset prices, because they depend on future endogenous behavior.

Another approach is to think of a state as a description of every aspect of reality that matters to each decision maker, including both, exogenous and endogenous events. This will have to include other people's behavior and the decision maker's own thought. The problem with this view is that the definition of the state becomes circular. Radner (1972) modeled uncertainty based on an intermediate position, namely, assuming that states are exogenous but assuming *common* expectations (that is, all traders associate the same prices to the same events.) Dutta and Morris (1997) have highlighted the fact that this entails both, complete agreement and degenerate beliefs with respect to the connection between (exogenous) events and prices. In this sense, it constitutes a partial abandonment of the hypothesis of subjective beliefs and, on the other hand, negates endogenous uncertainty –that is, uncertainty from the market process itself– for all unknowns are associated to the external events. Arrow and Hahn (1999) have also pointed to this as one of modeling choices that call for a revision. This line of research remains relatively unexplored yet.

It is possible that some events do not affect payoffs directly but may still affect beliefs (for instance, some moves from nature like sunspots). We will call them "pure informational events" to distinguish them from the "real events" which we have considered so far. Then, let us think of a state $\omega \in \Omega$ as being composed of two parts, $\omega = (\theta, \eta)$, where θ is a specification of the history of payoff-relevant actions whereas η is of payoff-irrelevant actions.

In what follows, it will be useful to recall a few definitions.

Definition 2 *A set H is called a **partition** of another set Ω iff $\cup_H h = \Omega$ and $h \cap h' = \phi \; \forall h, h' \in H$.*

Let us denote by $h(\omega)$ the element of H that contains ω.

Definition 3 *Let H and H^* be two partitions of Ω. H is said to be **finer** than H^* (denoted by $H \Subset H^*$) if $h(\omega) \subseteq h^*(\omega)$ for all $\omega \in \Omega$.*
If H is finer than H^*, then H^* is said to be coarser than H.

Notice that the relation $\Subset$ induces a partial order on the set of partitions of Ω, and therefore the supremum and infimum are meaningful.

Definition 4 *For any pair of partitions, their **join** is $H \vee H' = \sup \{H, H'\}$ and their **meet** is $H \wedge H' = \inf \{H, H'\}$.*

Thus, the join is the coarsest common refinement, and the meet the finest common coarsening. Events, then, are the elements of partitions $h_t(\omega) \in H_t$, which has the interpretation that each agent knows at time t that the true state is one of the elements of $h_t(\omega)$, among which he cannot recognize the true one —remaining uncertainty— and that he also can safely discard any $\omega' \notin h_t(\omega)$. The set of events will form a tree[4] represented by a sequence of partitions of Ω, $\{H_t\}_{t=0}^{T}$, where H_t is finer than H_{t-1} (representing the increased knowledge).

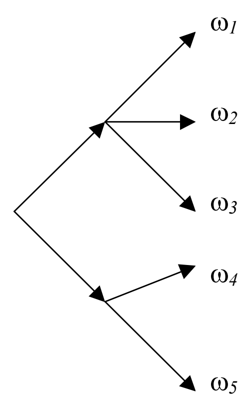

Figure 1: The event tree.

Figure 1 illustrates a situation where there are five states, each being a specification of, say, the actions of two persons at $t = 0$ and the choice of nature at $t = 1$. The states are given by $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$, and the tree can be

[4] A tree is a set of nodes such that each node has a unique predecessor. This is necessary to get a unique association between a state and a history, for if a terminal node had more than one backward path, the knowledge of the terminal node would not suffice to uniquely pin down what actually happened.

represented by the partition sequence $\{\{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}, \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4, \omega_5\}\},$ $\{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_4\}, \{\omega_5\}\}\}$.

Contingent claims are now related to events and not to states, and the same holds for contingent payments of assets, either pure or ordinary.

As for beliefs, rationality requires them to be updated according to Bayes' rule, that is,

$$\pi^i_{h_{t+1}} = \begin{cases} \frac{\sum_{\theta \in h_{t+1}} \pi^i_\theta}{\sum_{\theta \in h_t} \pi^i_\theta} & \text{if } h^i_{t+1} \subseteq h^i_t \\ 0 & \text{otherwise} \end{cases} . \tag{7}$$

Once these modifications are properly accounted for, everything remains the same, namely, the complete-market economy is Pareto efficient and there is no scope for speculation unless the completeness is achieved by retrading of long-lived assets.

2.3 Incomplete markets

Radner (1972) also presents what is nowadays the standard framework to analyze incomplete-market symmetric-information economies, that is, cases where "at every date and for every commodity there will be some future dates and some events at those dates for which it will not be possible to make current contracts for future delivery contingent on those events". In this setting, there is a nontrivial role for sequential trading because the opportunity of retrading the same assets is valuable as it enlarges the budget set. Hence, retrading acts as a substitute for inexistent markets[5].

Hirshleifer (1975) exemplifies this idea. His model corresponds to a two-good economy in which there are two rounds of trade, before and after the arrival of (private[6]) information, and consumption occurs at the last date. However, markets are incomplete since there are contingent claims for only one good; the second good must be traded *incontingently*. In the anticipation of a price change, individuals would trade to move from their endowment to a trading position, while they would trade again to go to the consumption position once the uncertainty is resolved. The incompleteness creates the need for trading in the second round, for as we have seen, under a complete market regime consumers could choose directly the final consumption bundle of contingent commodities, and any differences in beliefs would be reflected on date-zero prices. In fact, as pointed out in Hirshleifer (1977), "in the prior round each trader would be able to buy a portfolio covering his desired consumption baskets in the light of the alternative possible information-events as well as over the different state-contingencies".

Thus, the intuitive conclusion that speculation occurs owing to differing anticipations of price changes (and cannot be a consequence of a redistribution of risks) holds

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[5] This idea is further developed in Duffie and Huang (1985).

[6] Although the text considers the possibility that the information received by each trader is different, the model makes no special treatment of it. Moreover, since priors differ, posteriors would differ too, even though the information received be the same.

because price changes are a necessary condition for completing the market via contingent trading, i.e., to substitute for missing markets. Rubinstein (1975) expands on this idea by considering a three-date Arrow-Debreu economy, where a complete set of real-event contingent claims is available at every date but no information-event contingent claim and there is only one consumption good. In this setting, Pareto-optimallity obtains because retrading again substitutes for market-completeness.

Grundy and McNichols (1990) show that if it is known in advance that the market will be open in the future, it is not clear at all that agents would prefer *a priori* to trade in any of the available rounds in particular. In other words, if the market is known to reopen, it might be very active in the second date, but it is only so because it is necessary to complete transactions that could have been done in the first round but just weren't. Therefore, giving a nontrivial role to future rounds of trade requires the incompleteness of the set of available markets.

Up to this point, we have seen that existing asset markets allow individuals to smooth consumption and/or gamble. Speculation —trading and retrading based on belief heterogeneity— arises only if the market structure is incomplete, and has to be seen as the natural consequence of the need to substitute for missing markets. The next section describes how these conclusions change when belief heterogeneity is explained solely by the existence of private information.

3 Information-based trading

So far we have assumed belief heterogeneity without paying special attention to the sources of that heterogeneity. Indeed, the Walrasian tradition has been to treat beliefs as exogenous variables, just like utility functions. Each person then makes decisions based on observed prices without worrying about the origin of those prices because the information concerning why a relative price took a particular value — whether it can be traced back to beliefs, preferences, technology, etc.— is completely useless. An implicit assumption, then, is that information is symmetric, that is, the event tree $\{H_t\}_{t=1}^T$ is shared by everyone.

Yet, common wisdom points to differences in information as the main source of belief heterogeneity[7]. Speculators, in possession of more or better information, would be better-than-average forecasters earning a return for their social contribution in keeping prices in line with available information, which improves the quality of investments. This is in broad terms the view of Working (1953), and also what Fama (1970) had in mind when discussing the efficient market hypothesis.

One way to model information heterogeneity is to imagine that at every moment all information partitions differ, that is, everybody is informed of a different event. The informational structure is then defined as a state space Ω together with a collection

[7] Moreover, the "Harsanyi doctrine" holds that differences in information should be the only source of belief heterogeneity. See section 4.

of partitions of Ω, $\{H_t^i\}_{t=1,...,T}^{i=1,...I}$. Events are privately observed as long as $h_t^i \neq h_t^j$. What follows is a static analysis, so the time subscript is dropped.

Lintner (1969) presents what is possibly the first attempt to explicitly incorporate information as the source of belief heterogeneity. Using a general equilibrium model in which risk averse investors with exponential preferences, endowed with private signals $\{h^i\}_{i=1}^I$, have to make a portfolio decision between a risk-free and a risky asset whose return is normally distributed —the "normal-exponential model", which later became standard in the finance literature—, he shows that the equilibrium asset price depends on the vector of private signals. In particular, for an exogenous supply of the risky asset $\overline{x}$, its price at ω is given by

$$q(\omega) = \left(\sum_{i \in I} \frac{E\left[r\left|h_i(\omega)\right.\right]}{a_i Var\left[r\left|h_i(\omega)\right.\right]} - \overline{x} \right) \left(\overline{r} \sum_{i \in I} \frac{1}{Var\left[r\left|h_i(\omega)\right.\right]} \right)^{-1} \tag{8}$$

where r is the rate of return on the risky asset, and $\overline{r}$ its ex-ante expected value. This model seemed to confirm the long-held intuitions that asset prices aggregate information because demands are affected by the signals received, and hence asset prices are driven by information.

This model was criticized by the then novel theory of rational expectations, which asserts that if agents are rational, they should recognize that the way prices are formed makes them useful pieces of information in their own right. Grossman (1976), for instance, shows that in Lintner's model, the asset price is a sufficient statistic for all private signals, and hence provides a better guide to decision-making than each particular piece of information taken by itself. Thus, Grossman is able to show that private information is redundant once the price is known. This raises a logical drawback in Lintner's argument (namely, if every investor prefers to use the information contained in the price and discards his own signal, how can the price reflect that private information?), and casts doubts on his conclusions.

3.1 Rational expectations

The discussion that followed Grossman's article tried to see the generality of the argument and attempted to solve the paradox. The rational expectations approach was to assume that investors understand the way prices are formed, and that they consequently should use the information contained in those prices to revise their beliefs. Radner (1979) formalizes this idea in the following way. Let $q(\omega)$ be the price that would obtain in state ω. Construct the partition P of Ω in which each cell is given by all states compatible with that price: $P(\omega) = \{\omega' \in \Omega : q(\omega) = q(\omega')\}$. Then, the information agent i has is $H_i \vee P$ and not just H_i. Notice that $H_i \vee P$ is finer than H_i. We follow Dubey, Geanakoplos and Shubik (1987) in defining:

Definition 5 *A rational expectations equilibrium for the exchange economy* $\mathcal{E} = \{\Omega, (\mathbf{w}^i, U^i, H^i)_{i\in I}\}$ *is a pair* $(\mathbf{p}, \mathbf{x})$ *such that:*

1. $\mathbf{x}^i \in \{\arg\max U^i(\mathbf{x}^i)$ *subject to* $\mathbf{p}(\omega)' \mathbf{x}^i(\omega) \leq \mathbf{p}(\omega)'\mathbf{w}^i(\omega)\} \quad \forall i \in I$

2. $\omega, \omega' \in H_i \vee P \Rightarrow \mathbf{x}^i(\omega) = \mathbf{x}^i(\omega') \quad \forall i \in I$

3. $\sum_{i=1}^{I} x_{\theta\ell}^i = \sum_{i=1}^{I} w_{\theta\ell}^i \ \forall\theta \in \Theta_0, \ell \in L$

Radner (1979) proves that generically P is finer than $\vee_{i\in I} H^i$, that is, prices are fully revealing[8] (moreover, they can even convey information that nobody has!). This is to say, it was proven that Grossman's problem was indeed generic, and the paradox well alive.

With regard to speculation this result is very strong: information-based trading is completely ruled out because nobody can have better information than what is embedded in the price. The next subsection generalizes this impossibility to more general trading environments, showing that is not exclusive of rational expectations models.

3.2 Common knowledge and speculation

As we have seen, in the rational expectations approach the problem of keeping private information actually private arises from both, the influence of every trader on the price (no price-taking in an informational sense) and the knowledge every trader has of the price function. We will see that it is not the Walrasian bargaining feature of the model which drives the result, but the knowledge of others –their understanding

[8] There is an issue of cardinality though. The price function can be generically one-to-one with the set of states only if the set of states is not rich enough, otherwise partial revelation is obtained (Allen (1981)).

of the situation they are involved in–. We will first introduce the basic tools to model knowledge, then generalize the trading environment and conclude this section by showing that the previous result still holds.

If the information structure (that is, the collection of partitions of Ω, $\{H^i\}_{i\in I}$, representing the possible signals every player may receive) is commonly known[9], then knowledge of $h_t^i(\omega)$ also implies knowledge of what other agents may know, for individual i cannot reject the possibility that individual j knows $h^j(\omega')$ for any $\omega' \in h^i(\omega)$. Mutual knowledge is, then, implied this way by the state and the informational structure.

Define the knowledge operator as

$$K_i(E) = \{\omega \in \Omega : h^i(\omega) \subseteq E\} \tag{9}$$

which has the interpretation that player i knows that event E occurred if E cannot be ruled out in any of the states he considers as possible. Similarly, when individual i at ω cannot reject any of the states in which j knows E, then i knows that j knows E:

$$\omega \in K_i\left(K_j(E)\right) \tag{10}$$

Lengthier iterations of the knowledge operator reflect higher levels of mutual knowl-

[9] Aumann suggests this is indeed tautological, for part of the definition of the state is what each agent knows abouth the knowledge of others. Geanakoplos (1994) and Arrow and Hahn (1999) discuss this issue.

edge.

Definition 6 *(Aumann, 1976) An event E is common knowledge if $\wedge_{i \in I} h_i(\omega) \equiv \mathcal{M}(\omega) \subseteq E$.*

Clearly, if $h_i(\omega) \subseteq E$ then agent i considers E to be true in every state he sees as possible. This is also true for every $i \in I$, since $h_i(\omega) \subseteq \mathcal{M}(\omega)$. Moreover, all iterations of the form $K_k\left(K_j(...K_i(\omega))\right)$ are also true, so mutual knowledge is true even in infinite regressions. Common knowledge is, in a sense, the least knowledge one can attribute to anybody. On the other hand, the maximum knowledge in society is represented by the join of $\{H^i\}_{i \in I}$.

We now represent trades as bets. Ultimately, when an income stream is chosen over another, one could say that there is an implicit bet over the likelihood of the states where income is increased. In fact, buying a share in the hope of a price rise is to bet on a price increase. More precisely,

Definition 7 *A bet is a function $b : \Theta \longrightarrow \mathbb{R}^I$ specifying for each state θ a vector of monetary payoffs $\mathbf{b}_\theta = (b^1_\theta, ..., b^I_\theta) \neq \mathbf{0}$ such that, $\forall \theta \in \Theta$, $\sum\limits_{i \in I} b^i_\theta = 0$. Player i is said to have bet \$$b$ on state θ if, whenever some state $\theta' \in \Theta \diagdown \{\theta\}$ occurs, $b^i_{\theta'} = -b$.*

Thus, a bet is a vector of transfers that must be agreed upon by all participants.

For instance, in the Arrow-Debreu model, it is possible to define the indirect utility function of consumption by

$$v^i(\mathbf{b}^i) \;=\; v^i(b_0^i, b_1^i, ..., b_\Theta^i) = \max_{\{\mathbf{x}^i\}} U^i(\mathbf{x}^i) \text{ subject to} \tag{11}$$
$$\left\{\mathbf{p}_\theta' \mathbf{x}_\theta^i \leq \mathbf{p}_\theta' \mathbf{w}_\theta^i + b_\theta^i\right\}_{\theta\in\Theta_0}$$

It is important to keep in mind, though, that implicit on it there are beliefs, and there is knowledge that provides a basis for them. The function $v^i(\mathbf{b}^i)$, then, evaluates income streams on the basis of preferences, beliefs, and endowments. In particular, $v^i(\mathbf{b}^i)$ can evaluate bets.

A bet $\mathbf{b} = (\mathbf{b}^1, \mathbf{b}^2, ..., \mathbf{b}^I)$ will be carried over only if all individuals agree to it, that is,

$$v^i(\mathbf{b}^i) \geq v^i(\mathbf{0}^i)\ \forall i \in I \tag{12}$$

One might wish to separate out the part of a trade is due to differences in beliefs from the part due to consumption smoothing. Let $\widetilde{\mathbf{b}}$ be the bet that would be carried over if all beliefs would coincide and agree to implicit state prices, that is, $\boldsymbol{\pi}^i = \widetilde{\boldsymbol{\pi}}\ \forall i \in I$. Such a state-contingent transfer would of course be justified by the structure of endowments and risk aversion. Then, $\widehat{\mathbf{b}}{\equiv}(\mathbf{b}{-}\widetilde{\mathbf{b}})$ would be due entirely to differences in beliefs. Speculation, in this sense, would be the act of betting on some states based on deviant expectations. For if beliefs would coincide, then there would be no speculation at all.

It can readily be seen that this is exactly what is done in a Walrasian economy where Arrow–securities are traded. Security markets give the opportunity of betting in the above sense, because of differences in beliefs, endowments, or risk aversion. The only special feature of Walrasian economies is the way in which the available bets are determined –which are given by $\left[b_0^i = -\sum_{k=1}^{K} q_k a_k \text{ and } b_\theta^i = \sum_{k=1}^{K} r_{\theta k} a_k\right]$– and the fact that $\mathbf{q}$ satisfies market-clearing[10].

The present analysis, then, has the advantage that it includes –but is not limited to– Walrasian economies, for it can serve as a characterization of any voluntary process of trade.

Milgrom and Stokey (1982) prove the following theorem[11]:

Theorem 2 *Suppose all traders are risk-averse, that the initial allocation is Pareto-optimal, that agents' prior beliefs are common, and that each player i observes the information conveyed by the partition H^i. If it is common knowledge at ω that* $\mathbf{b}$ *is a feasible trade and that each trader weakly prefers it to the zero trade, then every agent is indifferent between* $\mathbf{b}$ *and the zero trade. If all agents are strictly risk averse then* $\mathbf{b}$ *is the zero trade.*

Thus, Geanakoplos (1994), in an fine survey on common knowledge, asserts that "The main conclusion is that an apparently innocuous assumption of common knowl-

[10] This is a bet if one considers $b_\theta^i + b_0^i$ to be tha payment to individual i. Indeed, $\sum_{i\in I}\left(b_\theta^i + b_0^i\right) = \sum_{i\in I}\left(-\sum_{k\in K} q_k a_k^i + \sum_{k\in K} r_{\theta k} a_k^i\right) = \sum_{k\in K}\sum_{i\in I}\left(-q_k a_k^i + r_{\theta k} a_k^i\right) = \sum_{k\in K}\left(-q_k + r_{\theta k}\right)\left(\sum_{i\in I} a_k^i\right) = 0$ since k is a financial asset and because of market clearing.

[11] Aside from notational differences, their theorem was stated with a softer assumption, namely, concordant beliefs rather than common priors. See Morris (1994).

edge rules out speculation, betting, and agreeing to disagree". Specifically, he provides a proof of the following:

Theorem 3 *Let $(\Omega, (H_i, A_i, s_i)_{i \in I})$ be given, where Ω is a set of states of the world, H_i is a partition on Ω, A_i is an action set, and the strategy $s_i : \Omega \longrightarrow A_i$ specifies the action agent i takes at each $\omega \in \Omega$, for all $i \in I$. Suppose that s_i is generated by the decision rule $\psi_i : 2^{\Omega} \longrightarrow A_i$ satisfying the sure-thing principle*[12]*. (Thus $s_i(\omega) = \psi_i(h_i(\omega))$ for all $\omega \in \Omega$, $i \in I$.) If for each i it is common knowledge at ω that s_i takes on the value a_i, then there is some single event E such that $\psi_i(E) = a_i$ for every $i \in I$.*

In the context of this survey, the ψ_i function corresponds to the strategy that trader i follows (indicating what to do at every information set $h_i(\omega)$ she may find herself in). The theorem, then, establishes that the same action profile could have been obtained with symmetric information in an otherwise similar game. Hence, the informational asymmetry is not the explanation for the observed actions.

Special cases of the above theorem are Aumann's agreeing to disagree result and Milgrom and Stokey's no-trade theorem. In fact, the following example is provided by the latter to see the role of common knowledge in the no-trade theorem:

[12] A version of the sure-thing principle says that if $\psi(A) = \psi(B) = a$ and $A \cap B = \phi$, then $\psi(A \cup B) = a$. The fact that expected utility (according to Savage's (1974) axiomatization) satisfies the sure-thing principle is obviously not innocuos. The central point is really intertemporal consistency.

Example 1 *(Milgrom and Stokey, 1982) There are two payoff-relevant states,* $\Theta = \{\theta_1, \theta_2\}$. *Two players must simultaneously decide whether they accept or reject a bet in the following terms: if state* θ_1 *materializes, player 2 (she) pays \$1 to player 1 (he); if* θ_2 *occurs, the reverse payment is carried out. Before making a decision, however, each of them gets to see a private signal (information event) within the following sets:* $H_1 = \{\{\eta_1, \eta_2\}, \{\eta_3, \eta_4\}, \{\eta_5\}\}$ *and* $H_2 = \{\{\eta_1\}, \{\eta_2, \eta_3\}, \{\eta_4, \eta_5\}\}$, *which in fact are two distinct partitions of* $\Lambda = \{\eta_1, \eta_2, \eta_3, \eta_4, \eta_5\}$. *They have common priors on* $(\Theta \times \Lambda)$ *given by*

	η_1	η_2	η_3	η_4	η_5
θ_1	0.20	0.05	0.05	0.15	0.05
θ_2	0.05	0.15	0.05	0.05	0.20

Suppose the true message is η_3. *Should they bet? The answer is no if there is common knowledge of rationality. To see this, imagine first that the players are rational but they are not aware of their opponent's rationality. Then, each player computes the expected value of betting according to*

$$\begin{aligned} E[u_1|\{\eta_3, \eta_4\}] &= \frac{2}{3}(1) + \frac{1}{3}(-1) = \frac{1}{3} > 0 \text{ and} \\ E[u_2|\{\eta_2, \eta_3\}] &= \frac{1}{3}(-1) + \frac{2}{3}(1) = \frac{1}{3} > 0. \end{aligned}$$

As both are positive, they would accept. Moreover, if each of them knew that his/her opponent is rational too, they would not only check their own answer but also their opponent's answer, according to the information they could have received. Indeed, *player 1 knows that his opponent could have received either* $\{\eta_2, \eta_3\}$ *or* $\{\eta_4, \eta_5\}$*, but as*

$$\begin{aligned} E[u_2|\{\eta_2,\eta_3\}] &= \frac{1}{3}(-1)+\frac{2}{3}(1)=\frac{1}{3}>0 \text{ and} \\ E[u_2|\{\eta_4,\eta_5\}] &= \frac{20}{45}(-1)+\frac{25}{45}(1)=\frac{1}{9}>0, \end{aligned}$$

her acceptance does not tell him anything new and his original calculation is still the most accurate. Similarly, she observes that his behavior would be the same irrespective of having received messages $\{\eta_1, \eta_2\}$ *or* $\{\eta_3, \eta_4\}$*, since*

$$\begin{aligned} E[u_1|\{\eta_1,\eta_2\}] &= \frac{25}{45}(1)+\frac{20}{45}(-1)=\frac{1}{9}>0 \text{ and} \\ E[u_1|\{\eta_3,\eta_4\}] &= \frac{20}{30}(1)+\frac{10}{30}(-1)=\frac{1}{9}>0. \end{aligned}$$

However, the analysis would be different with one more level of knowledge of mutual rationality. Suppose that not only player 1 knows she is rational, but also that he knows that she knows that he is rational. In that case, when receiving message $\{\eta_3, \eta_4\}$ *he knows she could have received messages* $\{\eta_2, \eta_3\}$ *or* $\{\eta_4, \eta_5\}$*, which in turn implies she would consider cases where he receives messages* $\{\eta_1, \eta_2\}, \{\eta_3, \eta_4\}$*, or* $\{\eta_5\}$*. Then she knows, he reasons, that if he accepts she can safely assume the message was not* $\{\eta_5\}$*, since*

$$E[u_1|\{\eta_5\}]=\frac{5}{25}(1)+\frac{20}{25}(-1)=-\frac{3}{5}<0$$

But then he must conclude that she will reject the bet if she receives the message

$\{\eta_4, \eta_5\}$, *since*

$$E[u_2|\{\eta_4, \eta_5\} \backslash \{\eta_5\}] = \frac{15}{20}(-1) + \frac{5}{20}(1) = -\frac{1}{2} < 0.$$

Similarly, as she would reject the bet when the message is $\{\eta_1\}$ *and he knows it, he in turn would not accept when receiving* $\{\eta_1, \eta_2\}$ *as a message. But this leaves us with the message* η_3 *as the only candidate for simultaneous acceptance of the bet. As in fact they both know it, in this case we have*

$$E[u_1|\{\eta_3\}] = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0 = E[u_2|\{\eta_3\}]$$

where their expectations have converged (negating asymmetric information) and there are no gains from trade.

Remark 1 *All this complicated reasoning is embedded in the definition of Nash equilibrium. Once common priors on the possible states of the world are assumed, the assumption of common knowledge is invoked when looking at a Nash equilibrium of* *this game. The above example, for instance, has the associated normal form:*

$s_1 \setminus s_2$	*000*	*001*	*010*	*011*	*100*	*101*	*110*	*111*
000	$\boxed{\mathbf{0,0}}$	*0,0*	$\boxed{\mathbf{0,0}}$	*0,0*	*0,0*	*0,0*	*0,0*	*0,0*
001	*0,0*	*-.15,.15*	*0,0*	*-.15,.15*	*0,0*	*-.15,.15*	*0,0*	*-.15,.15*
010	$\boxed{\mathbf{0,0}}$	*.10,-.10*	$\boxed{\mathbf{0,0}}$	*.10,-.10*	*0,0*	*.10,-.10*	*0,0*	*.10,-.10*
011	*0,0*	*-.05,.05*	*0,0*	*-.05,.05*	*0,0*	*-.05,.05*	*0,0*	*-.05,.05*
100	*0,0*	*0,0*	*-.10,.10*	*-.10,.10*	*.15,-.15*	*.15,-.15*	*.05,-.05*	*.05,-.05*
101	*0,0*	*-.15,.15*	*-.10,.10*	*-.25,.25*	*.15,-.15*	*0,0*	*.05,-.05*	*-.10,.10*
110	*0,0*	*.10,-.10*	*-.10,.10*	*0,0*	*.15,-.15*	*.25,-.25*	*.05,-.05*	*.15,-.15*
111	*0,0*	*-.05,.05*	*-.10,.10*	*-.15,.15*	*.15,-.15*	*.10,-.10*	*.10,-.10*	*0,0*

where acceptance is represented by a 1 and rejection by a 0. The knowledge of the opponent's partition is implied by the knowledge of his/her strategy. This game has four pure-strategy Nash equilibria (in bold), none of them corresponding to the common wisdom that an auspicious private message will induce betting (strategy profile $\mathbf{s} = (110, 011)$ *in the example). Following theorem 3,* $\psi_1(\{\eta_3, \eta_4\}) = \psi_2(\{\eta_2, \eta_3\}) = \psi_1(\{\eta_3\}) = \psi_2(\{\eta_3\}) = 0.$

The common prior and common knowledge assumptions, embedded on rational expectations models, explain Tirole's claim that "speculation relies on inconsistent plans and is ruled out by rational expectations". It turns out that speculation is ruled out in many other trading environment satisfying these assumptions as well.

This result is problematic for a theory of price formation under uncertainty. A part of the finance literature chose to drop the assumption of common knowledge as a mechanism to circumvent the problem. In fact, most of the private information literature was constructed initially over a modified version of the rational expectations model, namely, the noisy rational expectations model. The modification consists of adding a random shock to the asset supply, often justified because of the impredictability of trading behavior of some traders –henceforth noise traders–. This amounts to exclude a fraction of the population from what is explainable to both, the modeler and the rest of the agents in the model –thereby breaking down common knowledge–.

Grossman and Stiglitz (1980), for instance, use the normal-exponential so modified to show that private incentives to information-gathering activities are restored because no individual can perfectly infer the signal vector just by looking at the price –provided that there is enough noise, and that information is costly–. This class of model was developed further and applied to analyze informational issues notably by Hellwig (1980), Diamond and Verrechia (1981), Admati (1985), Kyle (1985), and Admati and Pfleiderer (1986).

Although the noise trader approach proved useful for analyzing information acquisition and aggregation under different market structures, the exogenous behavior of some agents on which it relies is not entirely satisfactory, for it raises the doubt as to which feature is responsible for obtaining such an incomplete inference from the price. Behavioral irrationality? (that is, trading no matter what the price or the information) Limited inference capabilities, or bounded rationality of some kind? In other words, the source of noise trading is at least as obscure as the source of the differences in beliefs that these models tried to illuminate. Another approach is to drop the homogeneous prior assumption, that is, to consider private information along with intrinsically different views of the world as explanations for speculation.

4 Heterogeneous priors

With heterogeneous priors, even public information generates trade, for individuals arrive to different conclusions and "agree to disagree"[13]. In effect, the arrival of information h will make every rational player to updated her prior beliefs π_θ^i according to Bayes' rule, that is:

$$\pi^i(\theta\,|h) = \begin{cases} \frac{\pi_\theta^i}{\sum\limits_{\theta' \in h} \pi_{\theta'}^i} & \text{if } \theta \in h \\ 0 & \text{otherwise} \end{cases}.$$

which will be different across individuals in general, as $\pi_\theta^i \neq \pi_\theta^{i'}$, creating a gap among marginal rates of substitution and hence gains from trade.

Although it does not immediately follow that private information will also generate trade in any environment, for information asymmetries open the possibility of

[13] Provided, of course, markets are incomplete. This scenario actually corresponds to the model in section 2.

misrepresentation –that is, revealing private information through behavior may not be incentive compatible, as explained by Morris (1994)–, in general it will, at least in competitive environments. The following are partial equilibrium examples of it.

Harrison and Kreps (1978) develop a model with a continuum of risk-neutral investors that hold heterogeneous expectations over the dividend process a single security follows. In particular, they consider a continuum of individuals $i \in [0,1]$, each belonging to a limited number of types $\gamma \in \Gamma$. All individuals of a particular type hold the same prior π_0^γ over the set of histories Ω, but priors differ across types. Information (including the current dividend) is public, so that expectations are driven by the common event-tree $\{H_t\}_{t=0}^T$. That is, for a particular history ω, the sequence $E_\gamma\left[q_{t+1}\middle|\, h_t(\omega)\right]$ will in general be different for each type. The model is simplified by assuming that the wealth of each class is sufficiently large (possibly infinite) and that short sales are not allowed, so that the asset will always be hold by individuals of the highest time-t valuation type. Further, it assumes point expectations for the price process as Radner did: associated to each state there is a unique price. Disagreement about future asset prices is, then, uniquely characterized by disagreement about states, and therefore accounted for by prior heterogeneity. As a result, at every point in time, the equilibrium price must be (weakly) larger than what any particular investor might think the worth is —the difference being a bubble—. The possibility of reselling, together with intrinsically different views of the world, increases the value of the asset over what any type will be willing to pay if obliged to hold it forever[14].

Harris and Raviv (1993) and Kandel and Pearson (1995) present models along the same lines. The former considers a two-type economy with T periods and risk-neutral investors, in which every period a public signal is observed. The signal will be either high or low. In any case, both groups will disagree because of the difference in their priors[15]. In turn, the latter constructs a three-period two-type economy, where posterior beliefs are normally distributed –with different means and variances across types– and preferences are exponential. The restrictions imposed in both cases are justified as simplifying assumptions adopted to make the model amendable to testing, which is their main purpose.

The debate on whether heterogeneous priors is a sensible assumption or not does not appear to be very important in finance. The central issue being discussed is whether differences in beliefs should be explained exclusively on the basis of information –the Harsanyi doctrine– or not. From a logical perspective, the issue is not clearly settled yet (see Morris (1995), Gul (1998) and Aumann (1998) for arguments on both sides). However, for all practical purposes heterogeneous priors seems to be a sensible assumption. Indeed, even the most prominent advocate of homogeneous

[14] This actually corresponds to their—and Keynes'—defininiton of "speculation."

[15] When they say they assume common priors in the paper, they refer to the marginal over payoff-relevant events θ and not to the actual prior over states ω. As the conjugate posteriors are different (what they call "different models"), then their priors over Ω must differ.

priors, Robert Aumann, says in reference to information as the only source of belief heterogeneity that "when we say all information, we mean *all*: the schools the players attended, their childhood experiences, even their genes (which indirectly reflect the experience of previous generations)" (Aumann, 1998).

It is very difficult indeed to think of a problem in finance in which private information has this broad meaning; on the contrary, informational issues refer usually to a tiny part of *all* information. Consequently, priors with respect to the marginal informational difference will never be the same. And although it might still be true that speculation cannot be the consequence of private information alone (in the sense of the last piece of information received by itself), private information will never come isolated from an individual's whole history. Isolating it, then, lacks any practical interest.

Having said that, we have to qualify this argument by pointing out that there is a fundamental difference between assuming heterogeneous priors from the outset or assuming common priors together with different personal histories: in one case there is nothing to learn from other person's beliefs, while in the other there is –the history itself–. When seen this way, the argument in favor of heterogeneous priors is a practical one: asymmetric information models of trade modeled as subgames of whole trader's histories are not likely to be tractable.

Heterogeneous prior models not only have gained support from a theoretical perspective, but also from an empirical one. The thirst for this sort of model comes from observations like Ross' (1989), that "It is difficult to imagine that the volume of trade in security markets has very much to do with the modest amount of trading required to accomplish the continual and gradual portfolio balancing inherent in our current intertemporal models." Harris and Raviv (1993) and Kandel and Pearson (1995) present examples of that. However, one objection to this line of research is that prior beliefs are exogenous unobservable variables and as such it is very difficult to agree on a set of "acceptable degrees of heterogeneity." How different priors can be? In principle, it would seem that too many degrees of freedom are open to the researcher. But this is not really the case. For instance, if a decision-maker is confronted repeatedly with the same situation, where a lot of data is available, prior beliefs should be less heterogeneous as compared to situations that do not occur very often.[16]

Another approach consists of forcing prior beliefs to be consistent with the data in empirical applications. Kurz (1995)[17], for instance, develops a dynamic model with learning where individuals may hold any beliefs as long as they are not contradicted by the data. In the model, belief heterogeneity is only limited by what individuals

[16] An argument of this sort can be found implicitly in Keynes' (1937) discussion of long-term vs. short-term expectations, or in the de-refinement proposed by Fudenberg and Levine (1993) "self-confirming equilibrium."

[17] Kurz's model not only abandons the common prior but also the common expectations assumptions.

observe. When taking the model to the data, beliefs are given a particular form.

Instead of calibrating the heterogeneity from the data in each application, one may wish to restrict prior diversity from the outset, using for that purpose the accumulated knowledge about human beliefs in other sciences. A behavioral approach would be to construct a model whose features are consistent with a variety of regularities accumulated in experiments or in the psychology literature. De Bondt and Thaler (1994), for instance, argue in favor of constructing descriptive models from assumptions that are approximately true, such as overconfidence (that is, overestimating the reliability of our own knowledge), or loss aversion and framing (the fact that people are more affected by losses than gains and would react differently if a given decision problem is framed as loss avoidance than as a gains comparison).

One example of this approach is provided by Daniel, Hirshleifer and Subrahmanyam (1998), who construct a model based on the idea of overconfidence. Again we have two populations, one of risk-neutral informed overconfident investors, and the other of risk-averse uninformed investors. There are three dates; at the initial date there is a portfolio decision, at the intermediate date informed investors observe a private signal and may choose to rebalance their portfolio, and at the final date consumption takes place. Individuals are overconfident in the sense that they associate to the signal a higher level of accuracy than it really has, with the effect of making them overreact.

It must be emphasized that prior heterogeneity is not a synonymous for irrational beliefs. Heterogeneous priors produce beliefs which are inconsistent across individuals (and hence opens the opportunity of mutually beneficial trade), but each belief may very well be consistent with itself (i.e., that it be updated according to Bayes rule). An overconfident person in the previous paragraph is not an irrational one, but only a badly informed person (about the quality of her own knowledge). Irrational beliefs, however, will in general be inconsistent across individuals as well.

It is natural to ask whether the market will eliminate those "wrong beliefs" (either uninformed or irrational), that is, if it will select better-informed (or rational) investors, increasing their relative wealth. The answer to this question is not necessarily affirmative. For instance, De Long et al. (1991) show that a group of overconfident investors —that is, those who underestimate risks— may not only survive, but even eventually dominate the market. Indeed, they will risk more; as long as the market rewards risk-taking, its wealth may increase over the long run even up to the point of dominating the group of investors whose beliefs are correct, and despite of the fact that they are more likely to become ruined and that their overconfidence makes them consume more. Kyle and Wang (1997) raise an additional source of supremacy in the context of a duopoly game: overconfident investors may outperform rational investors because overconfidence may act like a commitment device. Benos (1998), in effect, shows that overconfidence causes aggressive trading. In summary, even though the answer will in general depend on the environment, these examples show that it is not at all clear that the argument of survival favors the homogenous priors assumption.

Finally, not only rationality may constrain belief heterogeneity, but also market completeness. Araujo and Sandroni (1999) show that when asset markets are complete, the existence of equilibrium requires that agent's posterior beliefs eventually become homogeneous, for otherwise there would be certain events to which some individual associate negligible probability while others do not. This creates almost arbitrage opportunities, which are ruled out by equilibrium.

5 Concluding remarks

Many difficulties arise when attempting to rationalize the notion of speculation – understood as belief-based trading–. First, while differences in beliefs explain exchange, that trade will only constitute speculation if financial asset markets are incomplete (or at most essentially complete). If there were at every time a complete set of contingent claims, there would be no need to buy assets with a view to resell them at a later date.

Second, private information alone can not explain speculative behavior if individuals interpret information in the same way (common priors). If private information motivates trade, it is because there are (enough) other traders with different motives simultaneously participating in the market. Their presence is necessary to prevent full revelation of private information and/or to provide rents to be appropriated by speculators. Purely speculative markets can not exist.

One alternative to avoid the no-trade theorem is to drop the common knowledge assumption. The popular approach of noisy rational expectations models did it by restricting common knowledge to a subpopulation, whereas the rest of the population, that is, noise traders, behaved in an unpredictable (and impossible to understand) way. The shortcut is useful for many purposes, but the conclusions are unlikely to be robust as explanations for speculation. Even though the common knowledge assumption might be too strong, the problem is that there is no obvious way of relaxing it, and consequently attempts in this direction are likely to be rendered as ad hoc. For this reason, any progress in this line should come from the interactive epistemology literature.

Finally, if individuals interpret information differently (heterogeneous priors), then even public information can motivate trade –provided that markets are incomplete–. In this view, financial markets not only process information, but also select "models." Moreover, this selection might be permanent, as it is not obvious a priori that the "wrong" models will be driven out of the market by the "correct" ones.

From an empirical standpoint, heterogeneous prior models appear to explain in a parsimonious way many of the regularities found in the data; regularities that, at the same time, are difficult to reconcile with the homogeneous prior assumption. However, in order for this approach to gain wider acceptance, it is necessary that some consensus is achieved with regard to the kind of heterogeneity that is acceptable in a model. We have seen some steps in this direction, with arguments arising from evolutionary game theory and psychology (behavioral economics). We expect to see many contributions from these areas in the near future.

References

[1] ADMATI, Anat R. (1985), "A Noisy Rational Expectations Equilibrium for Multi-Asset Securities Markets," Econometrica 53, 629-657.

[2] ADMATI, Anat R. and Paul PFLEIDERER (1986), "A Monopolistic Market for Information," Journal of Economic Theory 39, 400-438.

[3] ARAUJO, Aloisio and Alvaro SANDRONI (1999), "On the Convergence to Homogeneous Expectations when Markets are Complete," Econometrica 67, 663-672.

[4] ARROW, Kenneth J. (1963), "The Role of Securities in the Optimal Allocation of Risk Bearing," Review of Economic Studies 31, 91-96.

[5] ARROW, Kenneth J. and Frank HAHN (1999), "Notes on Sequence Economies, Transaction Costs, and Uncertainty," Journal of Economic Theory 86, 203-218.

[6] AUMANN, Robert J. (1976), "Agreeing to Disagree," Annals of Statistics 4, 1236-1239.

[7] AUMANN, Robert J. (1998), "Common Priors: A Reply to Gul," Econometrica 66, 929-938.

[8] BENOS, Alexandros (1998), "Aggressiveness and survival of overconfident traders," Journal of Financial Markets 1, 353-383.

[9] DANIEL, Kent, David HIRSHLEIFER and Avanidhar SUBRAHMANYAM (1998), "Investor Psychology and Security Market Under- and Overreactions," Journal of Finance 53, 1839-1885.

[10] DEBREU, Gerard, "Theory of Value, an Axiomatic Analysis of Economic Equilibrium," Cowles Foundation Monograph 17, 1959.

[11] DE BONDT, Werner F. M. and Richard THALER (1994), "Financial Decision-Making in Markets and Firms: A Behavioral Perspective," NBER Working Paper 4777.

[12] DE LONG, J. Bradford, Andrei SHLEIFER, Lawrence H. SUMMERS and Robert J. WALDMANN (1991), "The Survival of Noise Traders in Financial Markets," Journal of Business 64, 1-19.

[13] DETEMPLE, Jerome and Shashidhar MURTHY (1994), "Intertemporal Asset Pricing with Heterogeneous Beliefs," Journal of Economic Theory 62, 394-320.

[14] DIAMOND, Douglas W. and Robert E. VERRECHIA (1981), "Information Aggregation in a Noisy Rational Expectations Economy," Journal of Financial Economics 9, 221-235.

[15] DUBEY, Pradeep, John GEANAKOPLOS and Martin SHUBIK (1987), "The Revelation of Information in Strategic Market Games: A Critique of Rational Expectations Equilibrium," Journal of Mathematical Economics 16, 105-137.

[16] DUFFIE, Darrel and Chi-Fu HUANG (1985), "Implementing Arrow-Debreu Equilibria by Continuous Trading of a Few Long-Lived Securities," Econometrica 53, 1337-1356.

[17] DUTTA, Jayasri and Stephen MORRIS (1997), "The Revelation of Information and Self-Fulfilling Beliefs," Journal of Economic Theory 73, 231-244.

[18] FRIEDMAN, Milton: "Essays in Positive Economics," The University of Chicago Press, 1953.

[19] FUDENBERG, Drew and David K. LEVINE (1993), "Self-Confirming Equilibrium," Econometrica 61, 523-545.

[20] GEANAKOPLOS, John: "Common Knowledge," in Handbook of Game Theory with Economic Applications, vol. 2, North-Holland, 1994.

[21] GROSSMAN, Sanford (1976), "On the Efficiency of Competitive Stock Markets Where Traders Have Diverse Information," Journal of Finance 31, 573-585.

[22] GROSSMAN, Sanford and Joseph STIGLITZ (1980), "On the Impossibility of Informationally Efficient Markets," American Economic Review 70, 393-408.

[23] GRUNDY, Bruce and Maureen McNICHOLS (1989), "Trade and the Revelation of Information through Prices and Direct Disclosure," Review of Financial Studies 2, 495-526.

[24] GUL, Faruk (1998), "A Comment on Aumann's Bayesian View," Econometrica 66, 923-927.

[25] HARRIS, Milton and Artur RAVIV (1993), "Differences of Opinion Make a Horse Race," Review of Financial Studies 6, 473-506.

[26] HARRISON, J. Michael and David M. KREPS (1978), "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations," Quarterly Journal of Economics 92, 323-336.

[27] HART; Oliver D. and David M. KREPS (1986), "Price Destabilizing Speculation," Journal of Political Economy 94, 927-952.

[28] HELLWIG, Martin F. (1980), "On the Aggregation of Information in Competitive Markets," Journal of Economic Theory 22, 477-498.

[29] HIRSHLEIFER, Jack (1975), "Speculation and Equilibrium: Information, Risk and Markets," Quarterly Journal of Economics 89, 519-542.

[30] ________ (1977), "The Theory of Speculation under Alternative Regimes of Markets," Journal of Finance 32, 975-999.

[31] KANDEL, Eugene and Neil D. PEARSON (1995), "Differential Interpretation of Public Signals and Trade in Speculative Markets," Journal of Political Economy 103, 831-872.

[32] KURZ, Mordecai (1995), "Asset Prices with Rational Beliefs," mimeo.

[33] KYLE, Albert S. (1985), "Continuous Auctions and Insider Trading," Econometrica 53, 1315-1335.

[34] KYLE, Albert S. and F. Albert WANG (1997), "Speculation Duopoly with Agreement to Disagree: Can Overconfidence Survive the Market Test?," Journal of Finance 52, 2073-2090.

[35] LEVINE, David K. (1982), "Theory of Price Bubbles," UCLA Department of Economics Working Paper No 246.

[36] LINTNER, John (1969), "The Aggregation of Investors' Diverse Judgements and Preferences in Purely Competitive Security Markets," Journal of Financial and Quantitative Analysis 4, 103-124.

[37] MAKOWSKI, Louis and Joseph OSTROY (1995), "Appropriation and Efficiency: A Revision of the First Theorem of Welfare Economics," American Economic Review 85, 808-827.

[38] MILGROM, Paul and Nancy STOKEY (1982), "Information, Trade and Common Knowledge," Journal of Economic Theory 26, 17-27.

[39] MORRIS, Stephen (1994), "Trade with Heterogeneous Prior Beliefs and Asymmetric Information," Econometrica 62, 1327-1347.

[40] MORRIS, Stephen (1995), "The Common Prior Assumption in Economic Theory," Economics and Philosophy 11, 227-253.

[41] RADNER, Roy (1972), "Existence of Equilibrium of Plans, Prices, and Price Expectations in a Sequence of Markets," Econometrica 40, 289-303.

[42] RADNER, Roy (1979), "Rational Expectations Equilibrium: Generic Existence and the Information Revealed by Prices," Econometrica 47, 655-678.

[43] RUBINSTEIN, Mark (1975), "Securities Market Efficiency in an Arrow-Debreu Economy," American Economic Review 65, 812-824.

[44] SAVAGE, Leonard J. (1974), "The Foundations of Statistics," second revised edition, Dover Publications, New York.

[45] TIROLE, Jean (1982), "On the Possibility of Speculation under Rational Expectations," Econometrica 50, 1163-1181.

[46] ________ (1989), "Theories of Speculation," in Financial Markets and Incomplete Information, edited by Sudipto Battacharya and George Constantinedes, Rowman & Littlefield.

[47] WORKING, Holbrook (1953), "Futures Trading and Hedging," American Economic Review 43, 314-343.